

# ПОСТРОЕНИЕ ТРЕХЗНАЧНОЙ ФАКТОРНО-ВЕРОЯТНОСТНОЙ ЛОГИКИ НА ОСНОВЕ ТРЕХЗНАЧНОЙ ЛОГИКИ ЛУКАСЕВИЧА Я. Джаббарзаде В.М.

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**Аннотация:** используя трехзначное пропозициональное исчисление Лукасевича, это исследование разрабатывает факторно-вероятностное пропозициональное исчисление, адаптированное для экспертных интеллектуальных систем. Оно описывает, как каждый фактор влияет на операции системы с указанными вероятностями, которые могут быть положительными, отрицательными или нейтральными. На основании таблицы истинности любой формулы трехзначное пропозициональное исчисление Лукасевича, дан алгоритм вычисления фактор вероятностей этой формулы. Результаты предлагают модель, которая может быть расширена до  $n$ -значных логик, где  $n$  превышает три.

**Ключевые слова:** трехзначная логика Лукасевича, факторно-вероятностное пропозициональное исчисление, экспертные системы, интеллектуальные системы,  $n$ -значные пропозициональные исчисления.

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## Introduction

The paper investigates a novel model for deploying expert and intellectual systems, diverging from traditional models grounded in many valued probability theories. By reinterpreting three-valued propositional calculi, the study provides a fresh perspective on the implementation of expert and intellectual systems.

## Main Part

Many expert or intellectual systems are influenced by external factors  $A_i$  ( $i=1,2,\dots,m$ ) that are independent of one another, each with a certain probability:

- positively with probability  $p_i : P(t/A_i)=p_i$ ;
- negatively with probability  $q_i : P(f/A_i)=q_i$ ;
- neutrally with probability  $r_i : P(n/A_i)=r_i$ ;

These probabilities are predefined by the expert.

For the application of Łukasiewicz's three-valued logic to these systems, each  $A_i$  factor term will be interpreted as a factor consideration. The definition of a factor formula is:

- Each  $A_i$  ( $i=1,\dots,m$ ) factor consideration is a factor formula.
- If  $A$  and  $B$  are factor formulas, then  $(\neg A)$ ,  $(A \wedge B)$ ,  $(A \vee B)$ ,  $(A \supset B)$  are also factor formulas.
- Factor formulas are derived solely from rules 1 and 2.

Using the tables from Łukasiewicz's negation logic [1], we provide algorithms for calculating factor formulas for negation operations.

The factor formula for negation  $A_i$  is:

$A_i$ Factor's Logical Value	$\neg A_i$ Factor's Logical Value	Probability
t(true)	f(false)	$q_i$
f(false)	t(true)	$p_i$
n(neutral)	n(neutral)	$r_i$

Then,  $P(t/\neg A_i)+P(f/\neg A_i)+P(n/\neg A_i)=1$

Assume a factor formula  $A$  consists of variables  $A_1, \dots, A_n$  for  $2 \leq n \leq m$  and:

$P(t/A_j)=p_j$ ,  $P(f/A_j)=q_j$ ,  $P(n/A_j)=r_j$  with

$P(t/A_j)+P(f/A_j)+P(n/A_j)=1$  (for  $j=1,\dots,n$ ).

The Łukasiewicz truth table for  $A$  consists of  $3^n$  rows, and we number these rows  $1, \dots, 3^n$ . For each row  $i$ , we take the product  $z_i = k_1^i \cdot \dots \cdot k_n^i$ . Here:

- If the variable  $A_j$  (for  $j=1,\dots,n$ ) in row  $i$  takes the value t (true), then  $k_j^i=p_j$ .
- If in row  $i$ , the variable  $A_j$  takes the value f (false), then  $k_j^i=q_j$ .
- If in row  $i$ , the variable  $A_j$  takes the value n (neutral), then  $k_j^i=r_j$ .

We take the sum of the  $z_i$  for the rows where the formula  $A$  takes the value t (true) as the probability  $P(t/A)$ . If there are no rows where the formula takes the value t, then  $P(t/A)=0$ . Similarly, we calculate  $P(f/A)$  and  $P(n/A)$ .

for the f (false) and n (neutral) values, respectively. If there are no rows where the formula takes these values, then  $P(f/A)=0$  or  $P(n/A)=0$ , respectively.

Given that  $(p_1+q_1+r_1)=1, \dots, (p_n+q_n+r_n)=1$ , it follows that:

$$P(t/A)+P(f/A)+P(n/A)=(p_1+q_1+r_1) \cdot \dots \cdot (p_n+q_n+r_n)=1.$$

If  $A$  and  $B$  are logically equivalent, then their truth tables coincide, thus:

$$P(t/A)=P(t/B); P(f/A)=P(f/B); P(n/A)=P(n/B).$$

The obtained results have been presented at conferences [2] and [3].

The demonstration of the algorithm for the calculations of probabilities for the formulas  $A_i \vee A_j$  and  $A_i \wedge A_j$  has been shown in detail in the paper [2].

Note 1: The obtained results can be used in calculating the probabilities of factor formulas when creating expert and intellectual systems.

Note 2: Instead of factors affecting positively or negatively with given probabilities, we can consider other options, for instance, factors affecting strongly, weakly or indeterminately with given probabilities.

Note 3: The obtained results can be used for building reliable systems from unreliable elements

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